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B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Fourth Semester

Mathematics

Skill Based Subject — TRIGONOMETRY, LAPLACE
TRANSFORM AND FOURIER SERIES

(For those who joined in July 2017 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The value of $(\cos\theta + i\sin\theta)^n$ is _____

- (a) 0 (b) 1
(c) $\cos n\theta + i\sin n\theta$ (d) $\sin n\theta + i\cos n\theta$

2. If $\cos\theta + i\sin\theta = x$, then the value of $x + \frac{1}{x}$ is _____

- (a) $2\cos\theta$ (b) $2i\sin\theta$
(c) $2i\cos\theta$ (d) $2\sin\theta$

3. The value of $2\sinh x \cosh x$ is _____

- (a) 0 (b) 1
(c) $\cosh 2x$ (d) $\sinh 2x$

4. The value of $\log_e(x + \sqrt{x^2 + 1})$ is _____

- (a) $\sinh x$ (b) $\cosh x$
(c) $\sinh^{-1} x$ (d) $\cosh^{-1} x$

5. The value of $L(1)$ is _____

- (a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$
(c) $\frac{2}{s^3}$ (d) 0

6. The value of $L^{-1}\left[\frac{1}{(s+a)^2}\right]$ is _____

- (a) e^{-at} (b) $e^{-at}t$
(c) $e^{at}t$ (d) e^{at}

7. The value of $L(te^{-at})$ _____

- (a) $\frac{1}{s+a}$ (b) $\frac{1}{(s+a)^2}$
(c) $\frac{s}{s+a}$ (d) $\frac{s}{(s+a)^2}$

8. The value of $L^{-1}\left[\frac{2}{(s-a)^3}\right]$ _____

- (a) te^{at} (b) t^2e^{at}
(c) te^t (d) t^2e^t

9. The function $\tan x$ is periodic with period _____

- (a) 0 (b) 2π
(c) π (d) 3π

10. Which one of the following is an even function?

- (a) x (b) x^3
(c) $\sin x$ (d) $e^x + e^{-x}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Express $\frac{\sin 6\theta}{\sin \theta}$ in terms of $\cos \theta$.

Or

(b) Expand $\cos^6 \theta$ in series of cosines of multiples of θ .

12. (a) Prove that $\sinh^{-1} x = \log_e \left(x + \sqrt{x^2 + 1} \right)$.

Or

(b) Find $\log(1-i)$.

13. (a) Find $L(\sin^2 2t)$.

Or

(b) Find $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$.

14. (a) Evaluate $\int_0^{\pi} e^{-2t} \sin 3t dt$.

Or

(b) Find $L^{-1}\left[\frac{1+2s}{(s+2)^2(s-1)^2}\right]$.

15. (a) Express $f(x) = x$ as Fourier series in $-\pi < x < \pi$.

Or

(b) Obtain the half range sine series for e^x in $[0, 1]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Prove that $\frac{ah}{\cos \theta} - \frac{bk}{\sin \theta} = a^2 - b^2$.

Or

(b) Show that
 $\sin^3 \theta \cos^5 \theta =$

$$\frac{1}{2^7 (\sin 8\theta + 2 \sinh 6\theta \sin 4\theta - 6 \sin 2\theta)}$$

17. (a) If $\cosh u = \sec \theta$, show that
 $u = \log \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right)$.

Or

(b) Find the general value of $\log_{(-3)}(-2)$.

18. (a) Find (i) $L(\cos at)$ (ii) $L(\sinh at)$.

Or

(b) Prove that $L^{-1} \left[\log \frac{s+1}{s-1} \right] = \frac{2 \sinh t}{t}$.

19. (a) Solve the equation $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t$
given that $y = \frac{dy}{dt} = 0$ when $t = 0$.

Or

(b) Solve the equations $3 \frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$,
 $\frac{dx}{dt} + 4 \frac{dy}{dt} + 3y = 0$ given $x = 0 = y$ at $t = 0$.

20. (a) Explain the Fourier series for odd and even functions.

Or

(b) Prove that
 $|\sin x| = \frac{2}{\pi} - \frac{4}{\pi} \left(\cos \frac{2x}{3} + \frac{\cos 4x}{15} + \dots \right)$.